PROBLEM 3

1. Completing the programme and results

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Description automatically generatedThe missing constants were filled in based on different sources. *C\_D* was taken from the OpenRocket estimation, *A* was calculated the maximum radius of the rocket*, m\_w* was taken from the given data, *m\_p, T* and *t\_burn* were taken from the engine manufacturer data.

A screenshot of a computer

Description automatically generated with medium confidence

The Euler integration loop was then completed according to equations (16) and (17).

A picture containing text, clock

Description automatically generatedThe velocity and altitude at burnout were calculated simply as the velocity and altitude at point *t\_burn/dt.* To avoid an index error, both values were approximated to integers (which might yield a point index a bit different than the actual one; this can be neglected because of the large number of points.

Text

Description automatically generatedThe time at apogee and the total flight-time are times at zero velocity and zero altitude (with t>0) respectively. A ‘for loop’ with a reasonable range of checked ranges around the points of interest was created.

The final values obtained from the calculation above are as follows: Text

Description automatically generated

1. Comparison and correction of results

The results above are all (significantly) higher from the ones obtained in Problem 2. The most important cause of this is the approximation of β used there. However, in the programme, this coefficient is calculated using the geometric and aerodynamic properties of the rocket, as well as the air density at a given altitude. This is very significant, as at an altitude of 12000m this value is around 4 times lower than the sea level density, which will reduce the drag 4 times.

My first idea was to relate the K coefficient to the average mass during the engine burn and after the burnout (it was only related to the former initially). However this did not significantly change the obtained values.

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Description automatically generatedTo obtain convergence of the two results, the programmes’s accuracy has to be, in fact, decreased. To do this, the drag acceleration function was redefined.

This change yields the following values:

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Description automatically generated

The changed values are very close to the ones obtained in Problem 2. However, their accuracy is much lower than with the initial programme.

1. Maximum speed

The maximum speed was taken as the maximum value from the velocity list:



This is the velocity at burnout.

1. Dynamic pressure calculations

The dynamic pressure is defined as:

Based on this equation, a Python function for *q(y,v)* was defined. An analysis of the results for given altitudes and velocities yields the following maximum pressure and additional parameters at that point. It is worth noting that the dynamic pressure is maximum at the engine burnout.



1. Drag in terms of dynamic pressure

The drag can be defined as:

where is a unit vector in the direction of the velocity vector. In scalar form:

For a constant and the magnitude of drag will always be maximum at the maximum dynamic pressure. This will not be the case if the area or the drag coefficient changes ( will change because of the variation of, for example, the viscous and wave drag).

1. Apogee for launch at an angle

The simplest way of calculating the apogee at an angle of 75 degrees would be taking the already calculated apogee and multiplying it by the sine of the angle to get the vertical ‘component’ of the displacement, which yields an altitude of 11362 m. However, this method is not the most accurate because of the incorrect equations of motion for this case. To increase the accuracy, the equation of motion in the upwards direction should include only one component of the thrust and one component of the drag, so that:

To comute the new apogee altitude, the programme has to be changed by multiplying the thrust and drag accelerations by , which gives a new apogee of 12136 m. Modifying the equations from Problem 2 for the angle will yield a new apogee of 3410 m.

1. Assumptions made in the simulation

* **The flight is vertical only – this maximises the apogee makes for un unrealistic estimation**
* The wind gusts are neglected – this decreases the value of estimated drag and
* **The acceleration resultant from the thrust is constant – both the thrust curve and the mass vs time curve are non-linear, so the asumption will cause errors**
* **The rocket is flying in an incompressible regime – this significantly decreases the assumed value of drag and neglects other aerodynamic effects. In reality the rocket might fly at speeds close to Mach 2**
* The density of air is governed by the one equation regardless of altitude – the governing equation (for the ISA) changes after the 11000 m mark
* The lift force generated by the rocket is neglected – the calculations will have a small error
* The change with altitude of the gravitational acceleration is neglected – this is not significant for a low flight
* The drag coefficient is 0.54 – this is not an exact value and should be calculated more closely, the drag calculation will be inaccurate

The most important assumptions are highlited in **bold** and concern the vertical flight assumption, the thrust approximations and the assumed incompressibility of the flow.